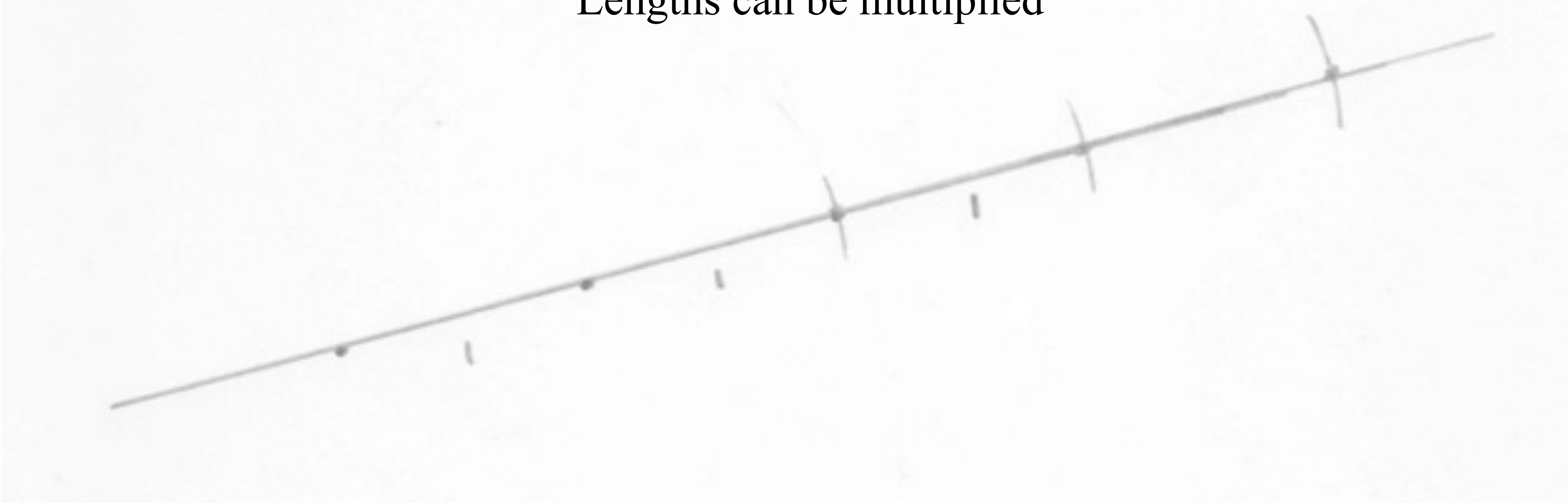
17 Sides

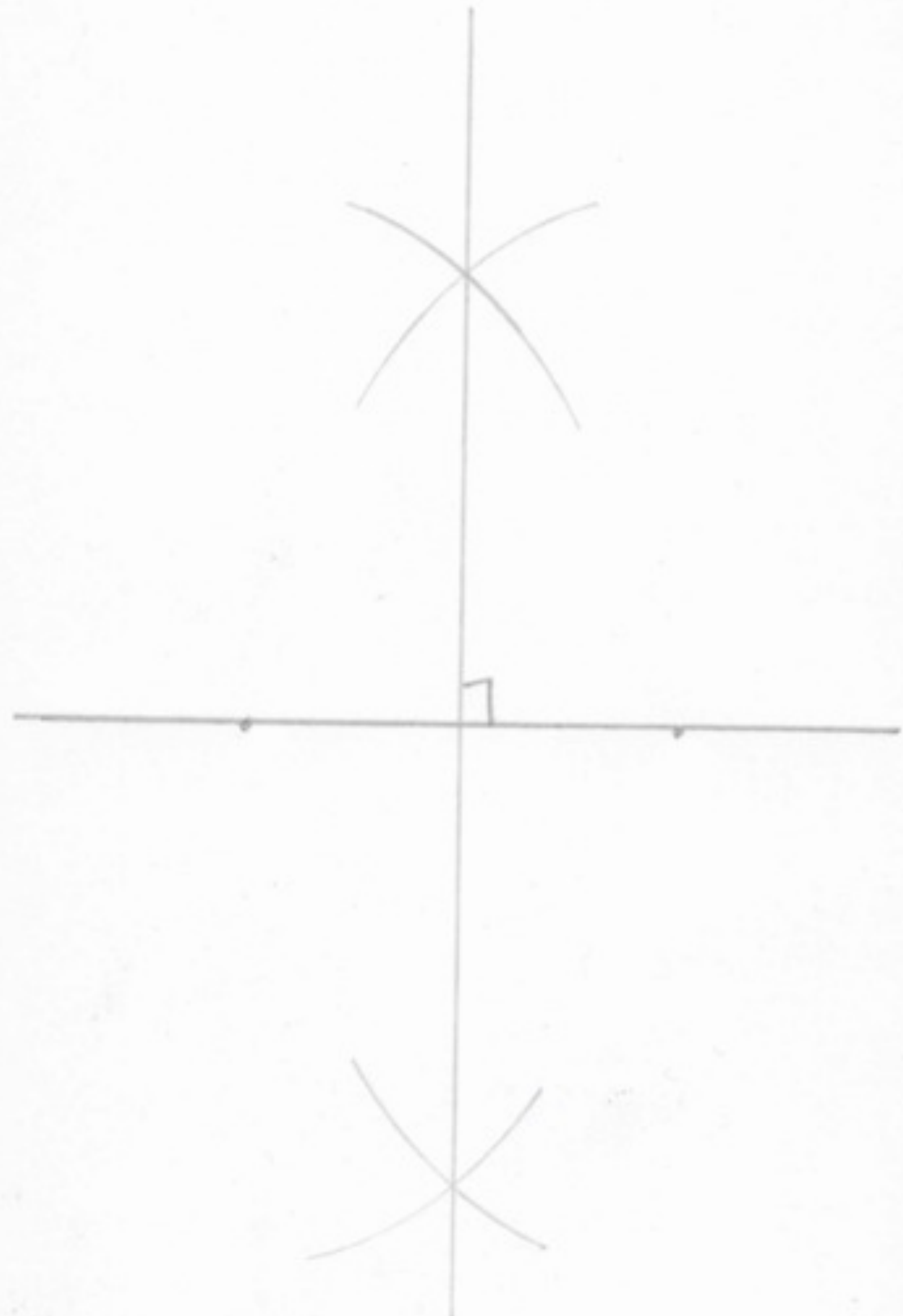
By Mike Frost

What Can You Construct with Straight Edge and Compass?

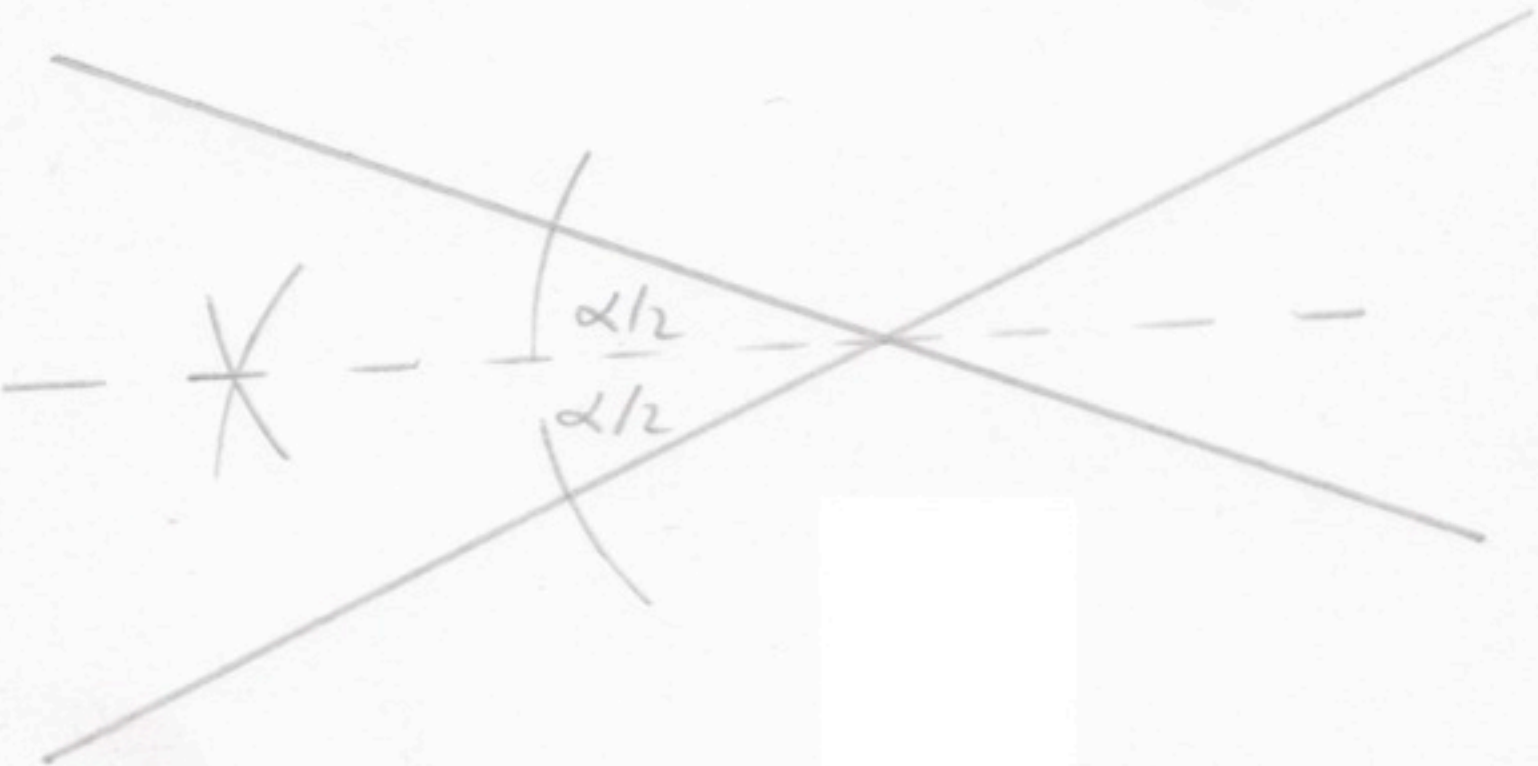
Lengths can be multiplied

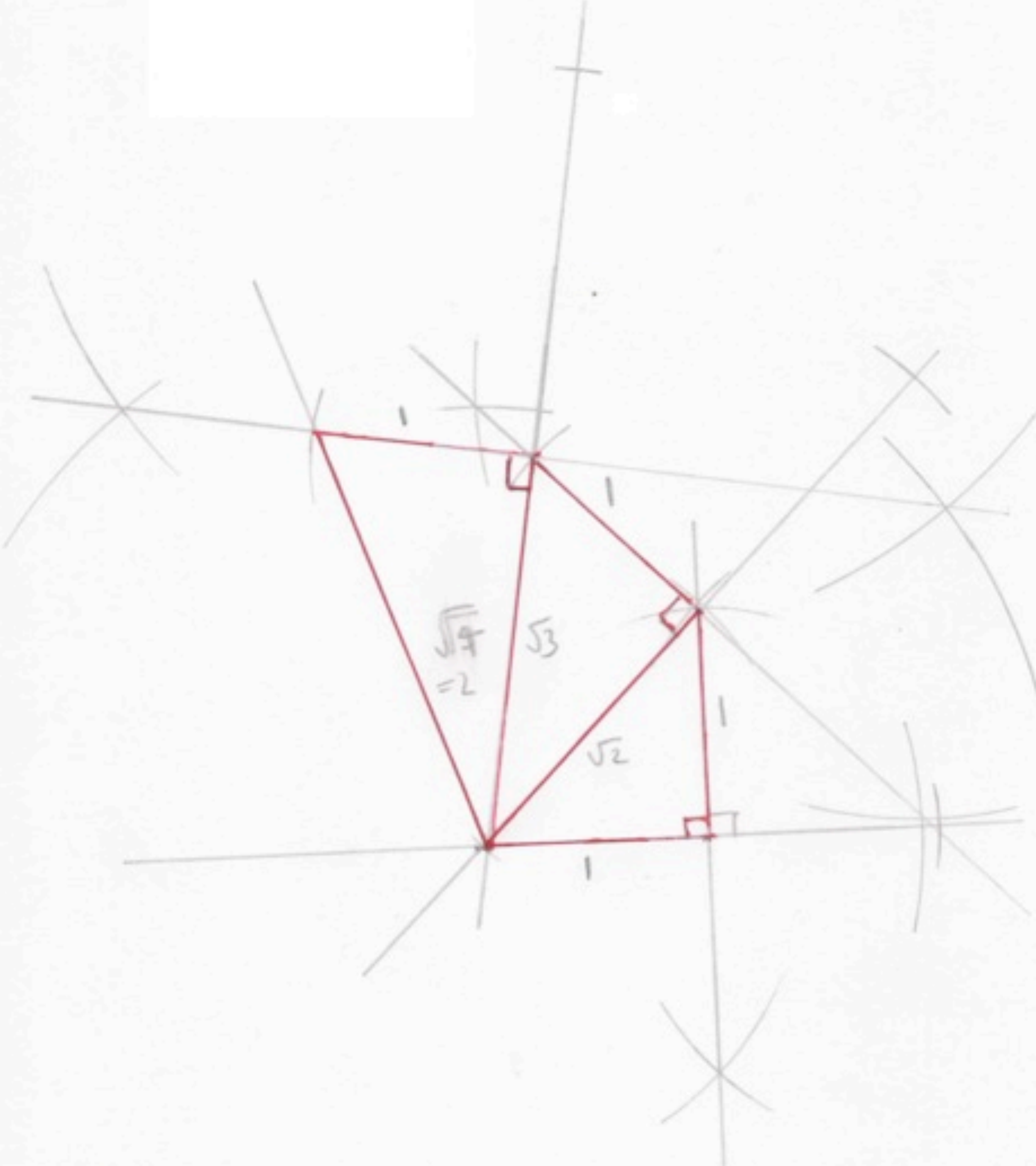


Lengths can be halved by Dropping a Perpendicular

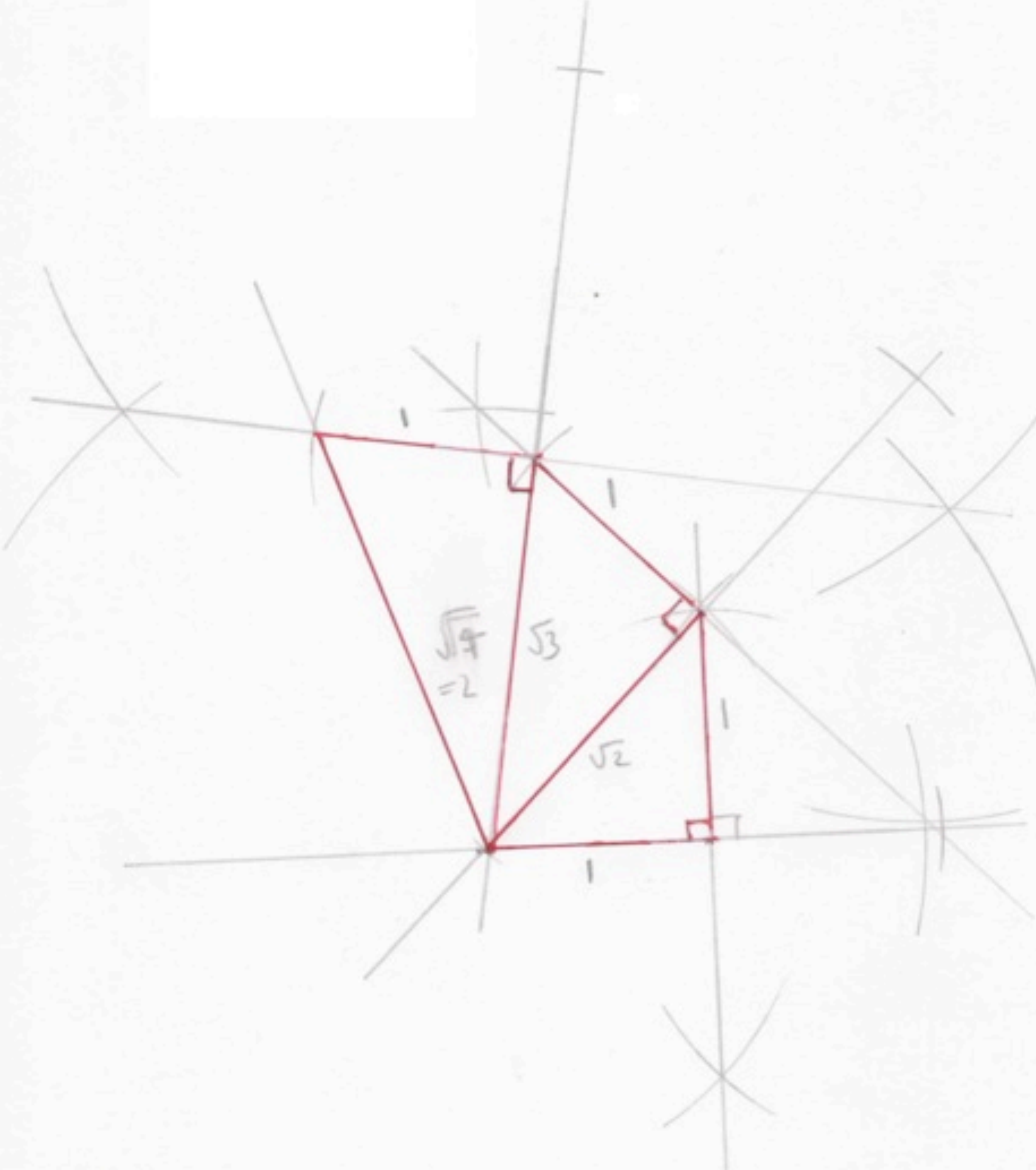


Angles can be Halved





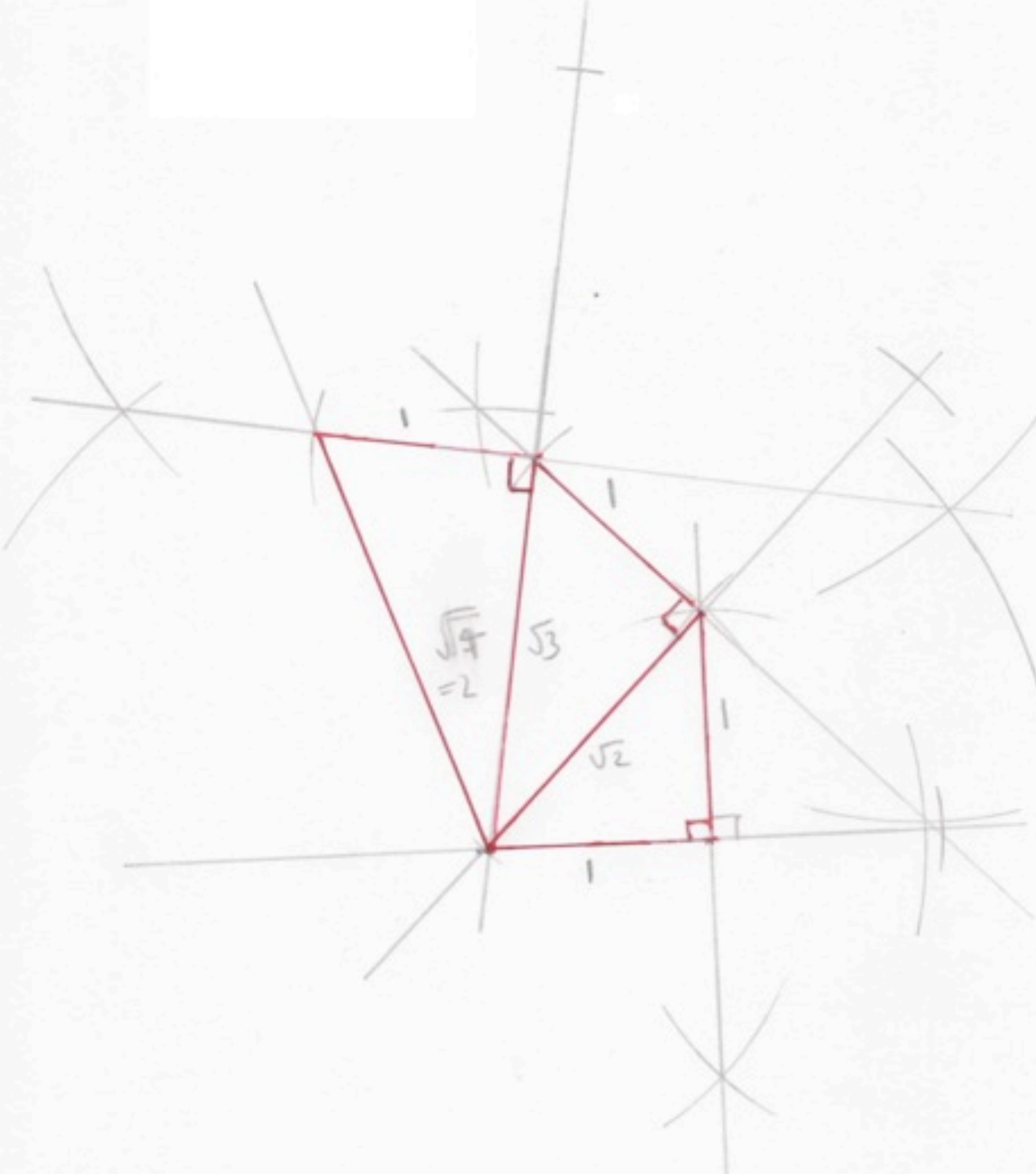
Square Roots can be taken



Square Roots can be taken

But you can't:

Trisect an Angle

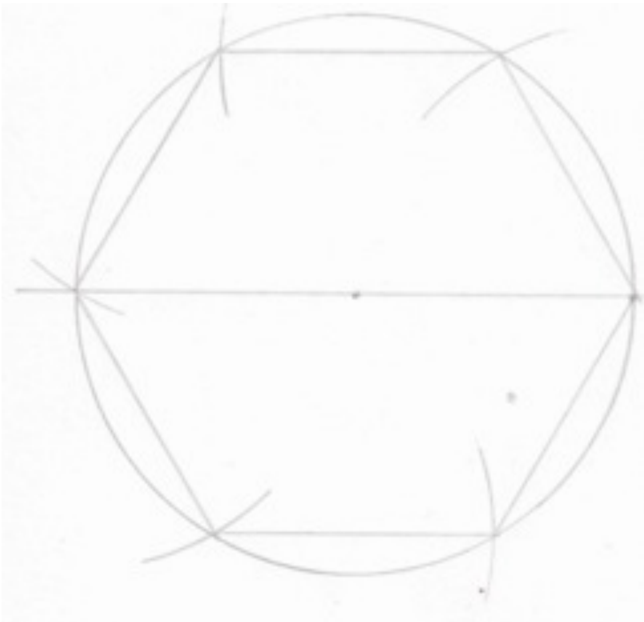


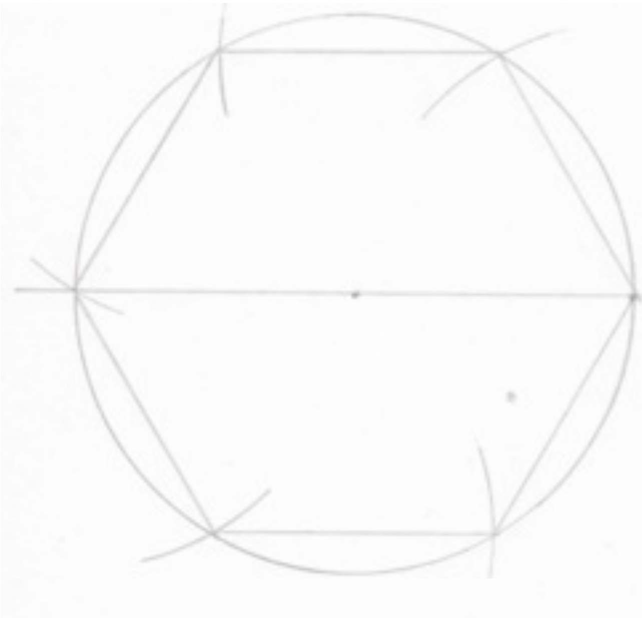
Square Roots can be taken

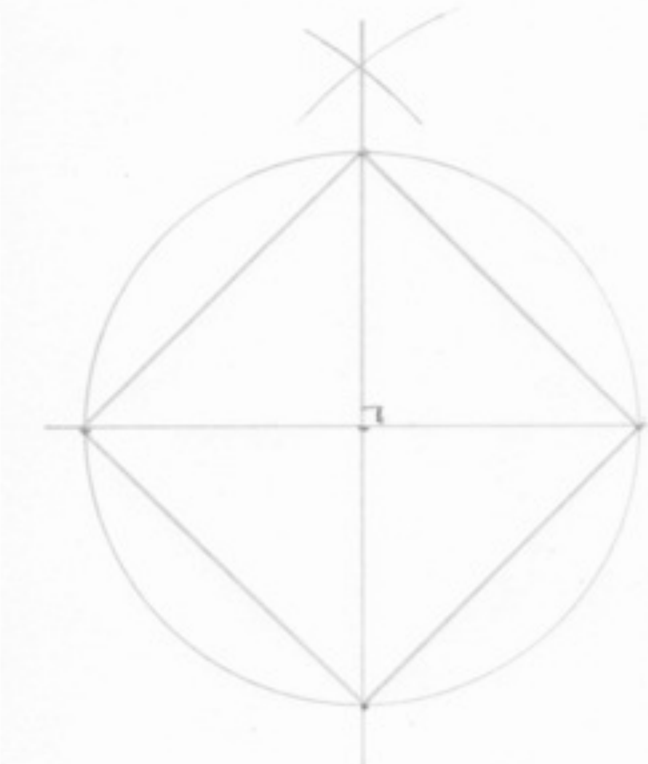
But you can't:

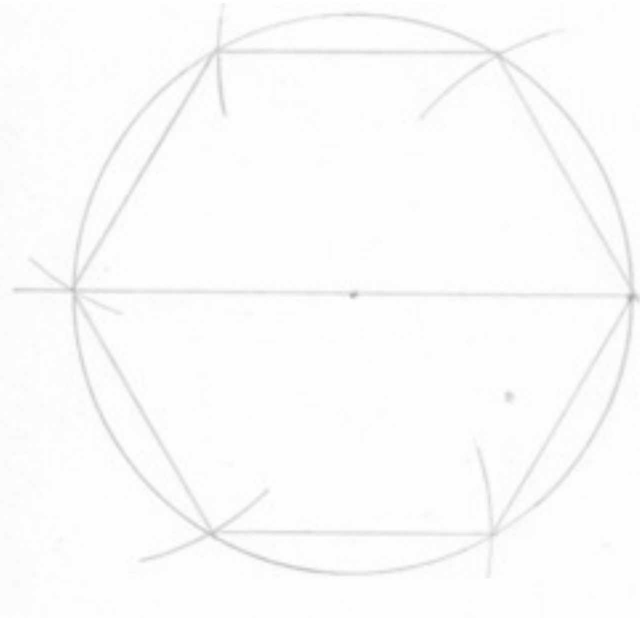
Trisect an Angle

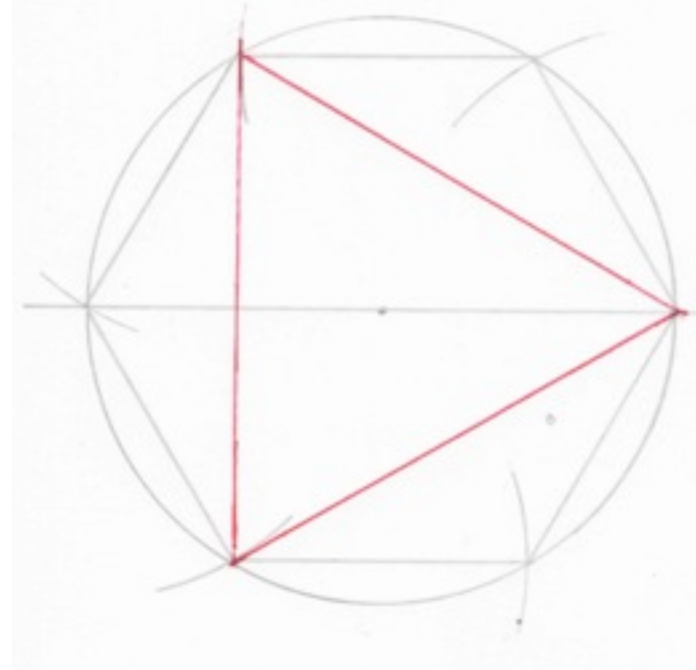
or take Cube Roots

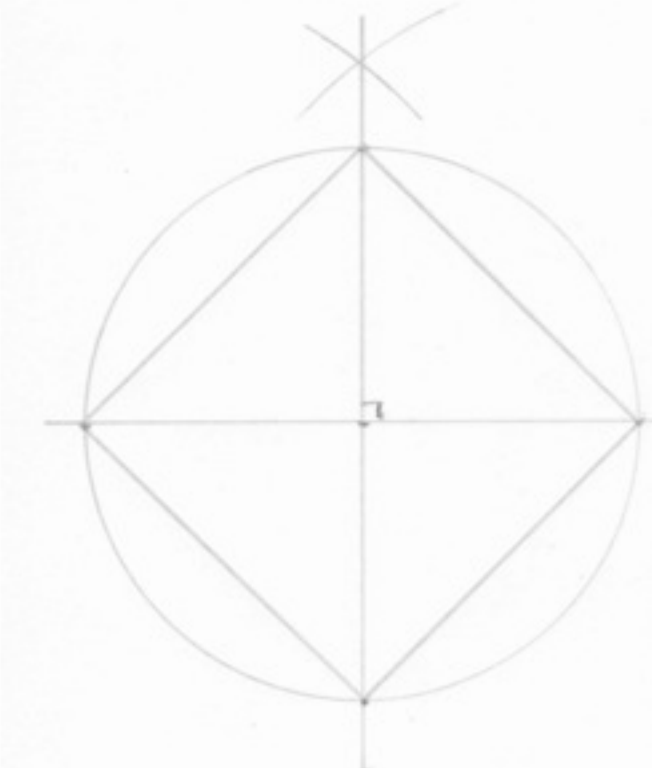


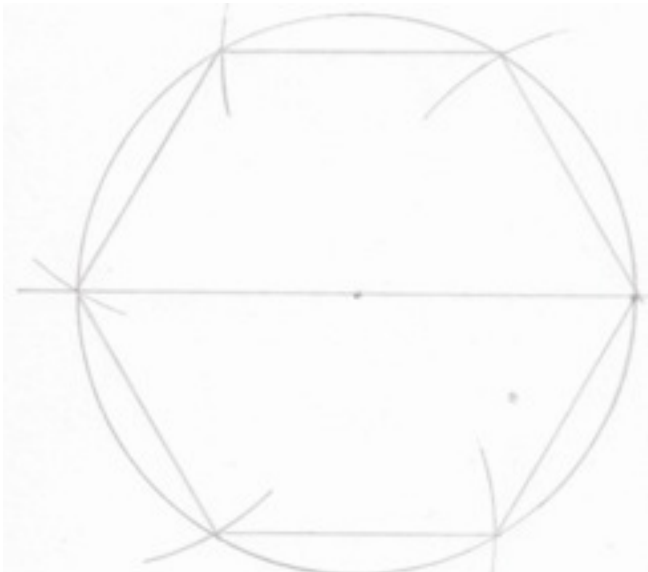
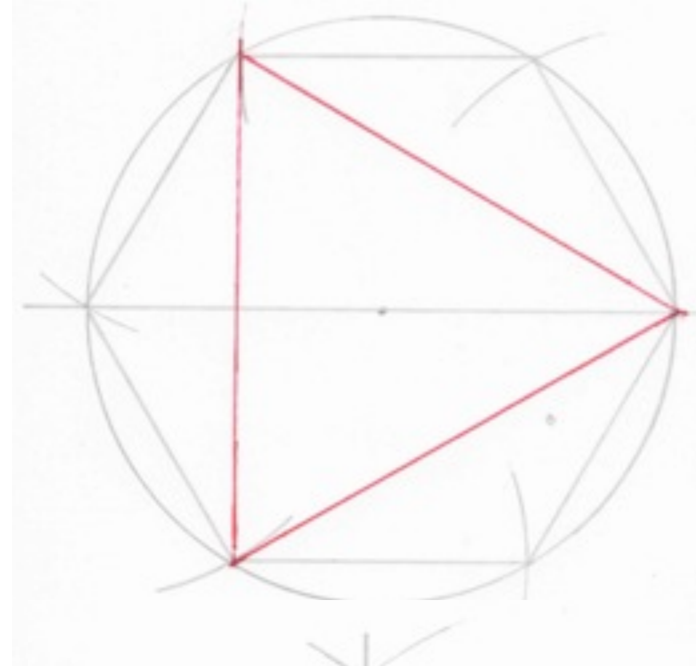
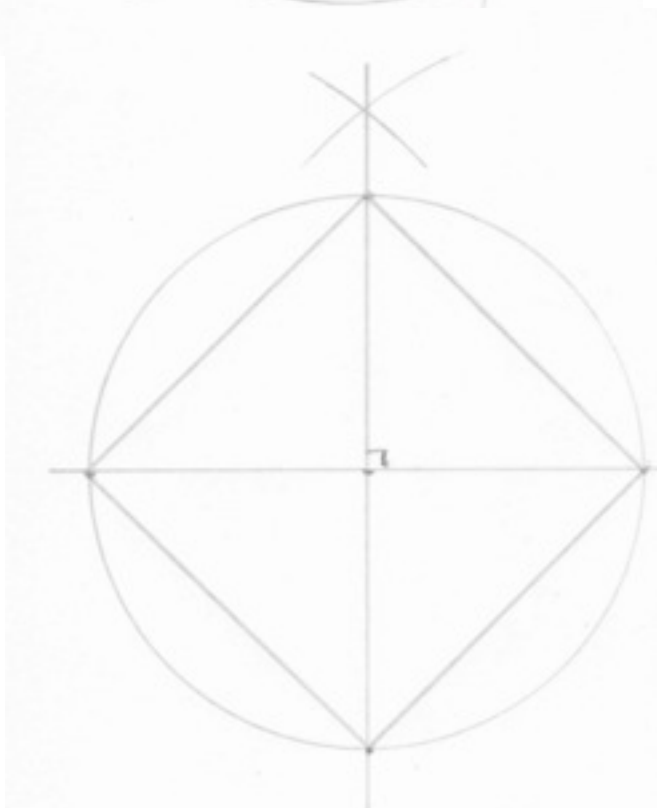
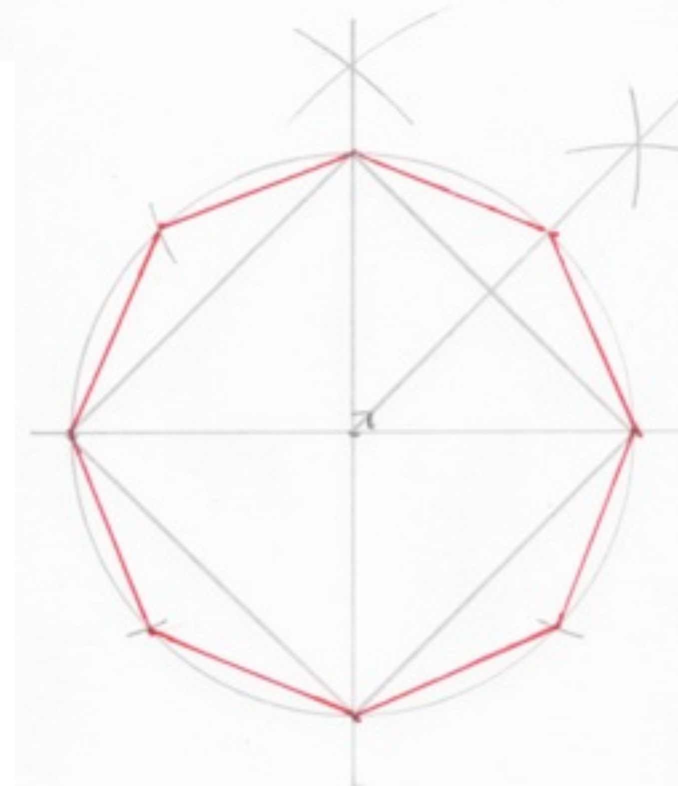










Constructing the

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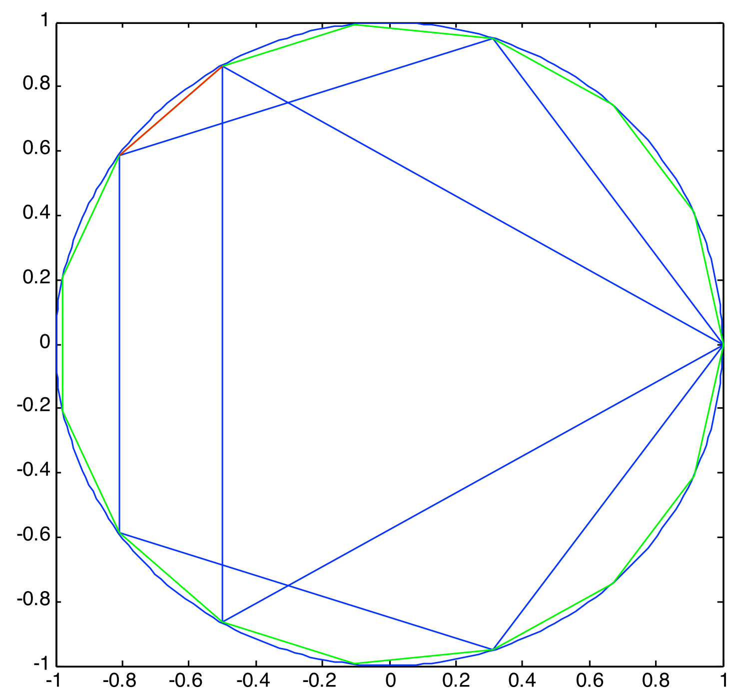

Carl Friedrich Gauss

To construct a Polygon with a prime number of sides n, n must be a Fermat Prime, of the form:

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Five known Fermat Primes N=0,1,2,3,4

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$$F_n = 2^{2^n} + 1$$

$F_0 = 3$

Five known Fermat Primes N=0,1,2,3,4

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Five known Fermat Primes N=0,1,2,3,4

$F_0 = 3$

$F_1 = 5$

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Five known Fermat Primes N=0,1,2,3,4

$F_0 = 3$

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$F_2 = 17$

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$F_4 = 65, 537$

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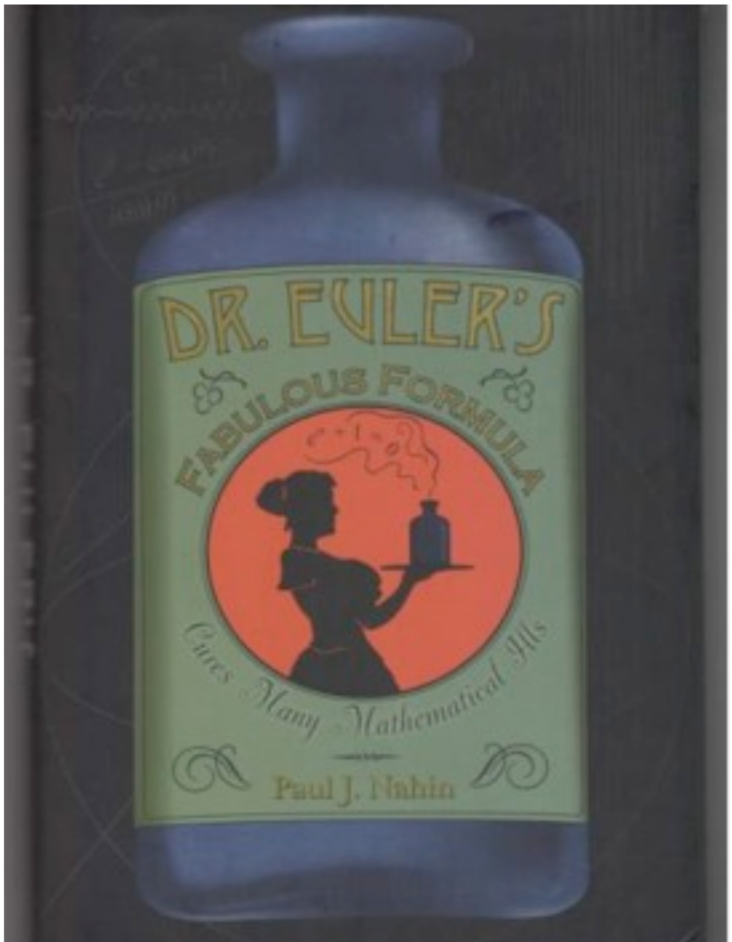
"Dr Euler's Fabulous Formula" – by Paul Nahin



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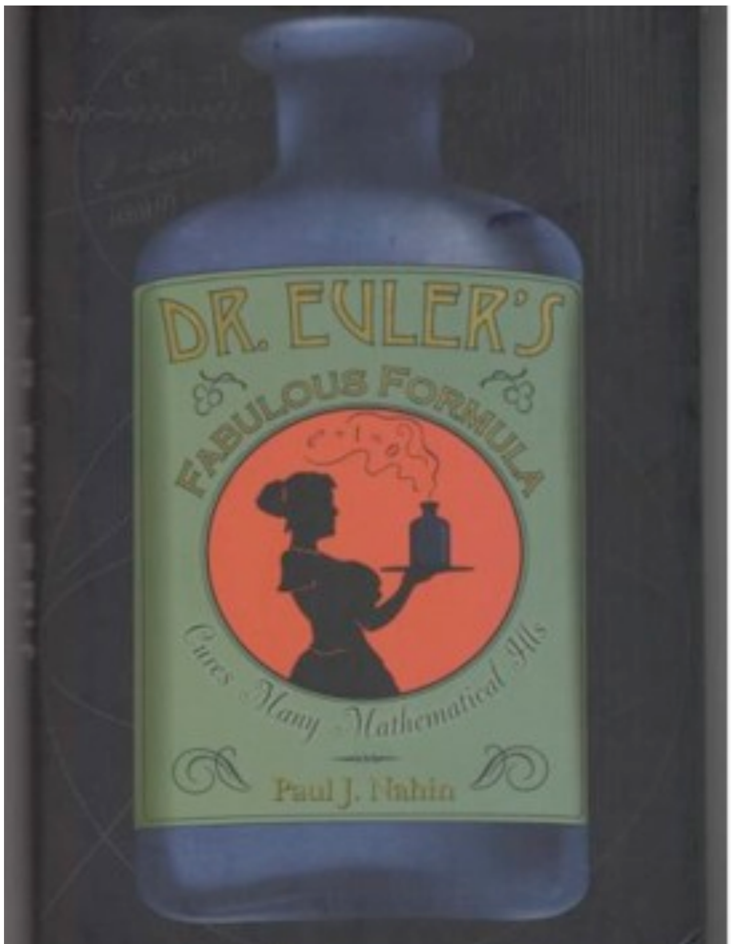

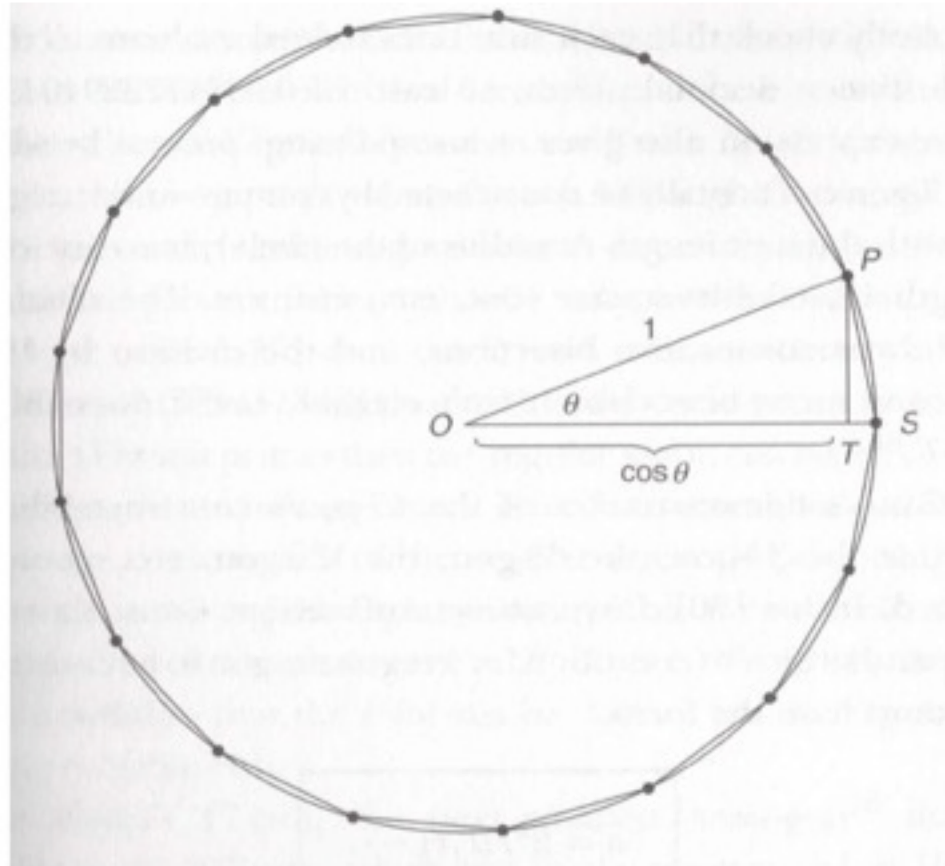

Figure 1.6.2. Gauss's regular 17-gon

$$\cos(2\pi / 3) = -0.5$$

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$\cos(2\pi / 17) \ldots$

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Cos (2 π / 17) ...

$$16\cos\frac{2\pi}{17} = -1+\sqrt{17}+\sqrt{34-2\sqrt{17}}+ 2\sqrt{17+3\sqrt{17}-\sqrt{34-2\sqrt{17}}-2\sqrt{34+2\sqrt{17}}}.$$

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Cos (2 π / 257) ?

CHRISTIAN GOTTLIEB

The Simple and Straightforward Construction of the Regular 257-gon

Regular Polygons

Most easily constructed of them all, and maybe the most beautiful also, is of course the regular hexagon. We all learned at school that the side of the regular hexagon equals the radius of its circumscribed circle, and this is really all one has to know.

Connecting every other vertex of the hexagon, we obtain an equilateral triangle (i.e., a regular 3-gon), and by repeatedly bisecting angles, we may obtain any regular $2^n \cdot 3$-gon, $n = 0, 1, 2, \ldots$. *Trisecting* angles, however, is not possible in general using the ruler and compass in the classical way, so, for example, the regular 9-gon is not constructible, but this was actually not definitely settled until the nineteenth century by Wantzel [9].

The construction of the regular pentagon is far less commonly known to nonmathematicians than that of the regular hexagon. It may be found in lots of textbooks on elementary geometry or history of mathematics; a good reference is the book by Aaboe [1]. The construction of the pentagon and its connections with the "golden section" were known already in ancient Greece.

The hexagon and pentagon lead directly to a construction of the regular 15-gon (this is simply because $\frac{1}{15} = \frac{2}{5} - \frac{1}{3}$). Analogously, the regular ab-gon can be found from the regular a- and b-gons whenever a and b are relatively prime. On the other hand, it may be shown that the regular p^2-gon is never constructible for p odd. For these reasons, investigations on regular p-gons may be confined to p-gons, where p is an odd prime number.

We have $p = 3$ and $p = 5$ so far. By the end of the eighteenth century, it was the common opinion among mathematicians that no other p-gons were constructible. Indeed, more than 2000 years had elapsed since the 3- and 5-gons were first constructed, and no other p-gon had appeared.

But on 30 March 1796, the young Carl-Friedrich Gauss wrote in his diary (see [3]) that he had found "the principles on which the partition of the circle is based, and in particular the geometrical divisibility of the same into seventeen parts" (this is a free translation from Latin). In other words, he had found that the regular 17-gon is constructible. Naturally, he was very pleased with this discovery, and it is said that it influenced him strongly to devote his life to mathematics.

Why a 17-gon? What do $p = 3$, 5, and 17 have in common more than being prime numbers? First, note that when we inscribe a p-gon in a circle, we start with one first vertex, which we may choose arbitrarily, and our task is to find the $p - 1$ "unknown" vertices. Therefore, along with p, the number $p - 1$ seems rather relevant to the situation. For p as above, we get $p - 1 = 2$, 4, and 16, respectively,

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...and Cos (2π/ 65,537) ??

The Biggest Constructible Odd-Numbered Polygon

3 . 5 . 17 . 257 . 65537

= 4 , 294 , 967 , 295 Sides

1

Constructing the 17-Gon