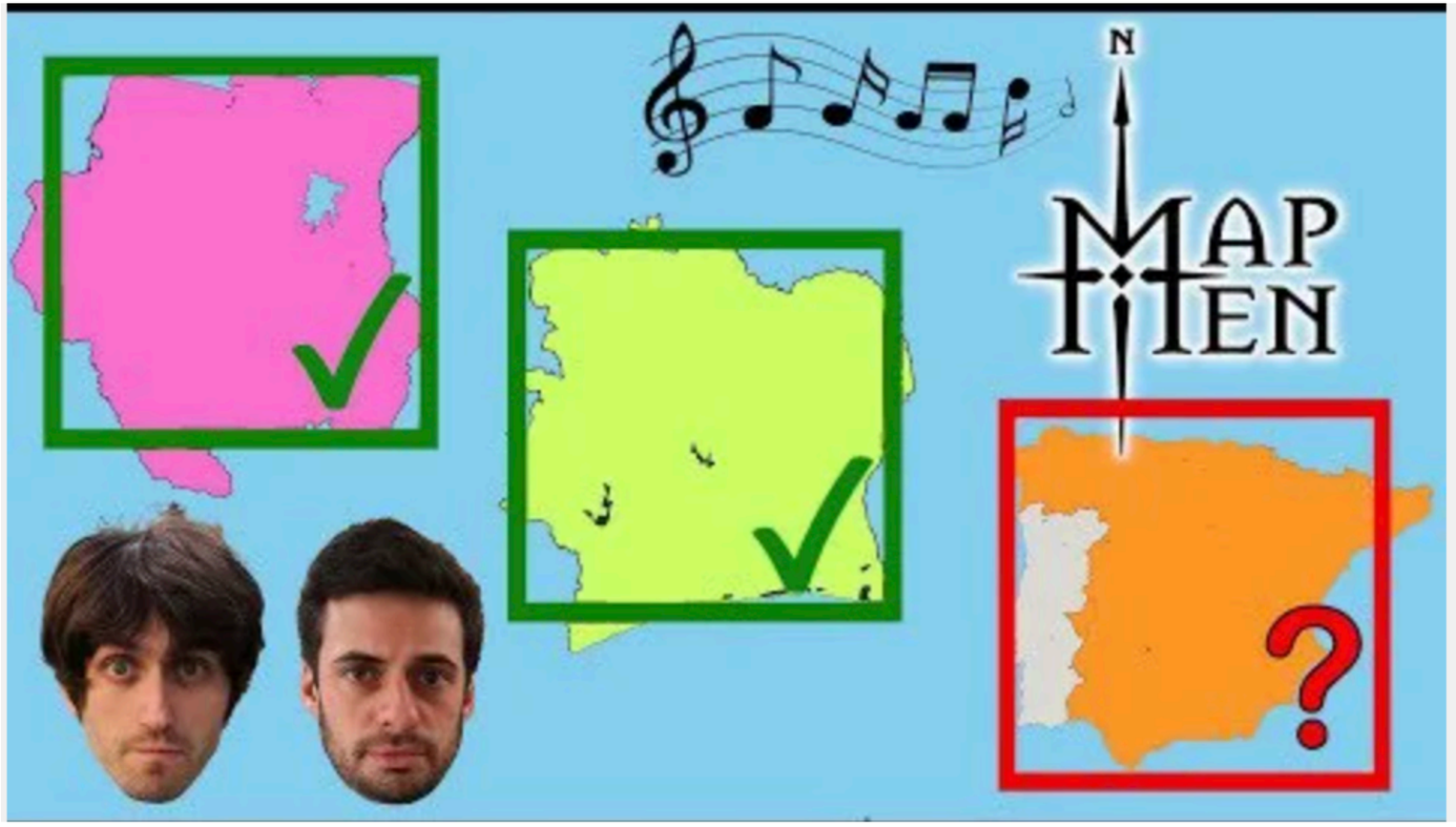
How square is a circle

Not very! But really, how square?

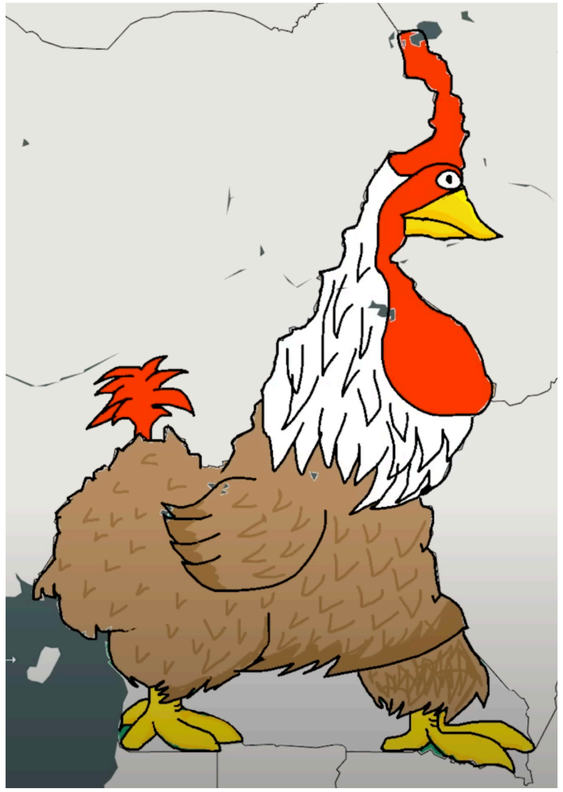
Ben Handley, MathsJam 2023

The mystery of the squarest country



Countries that are not at all square

UK



Cameroon

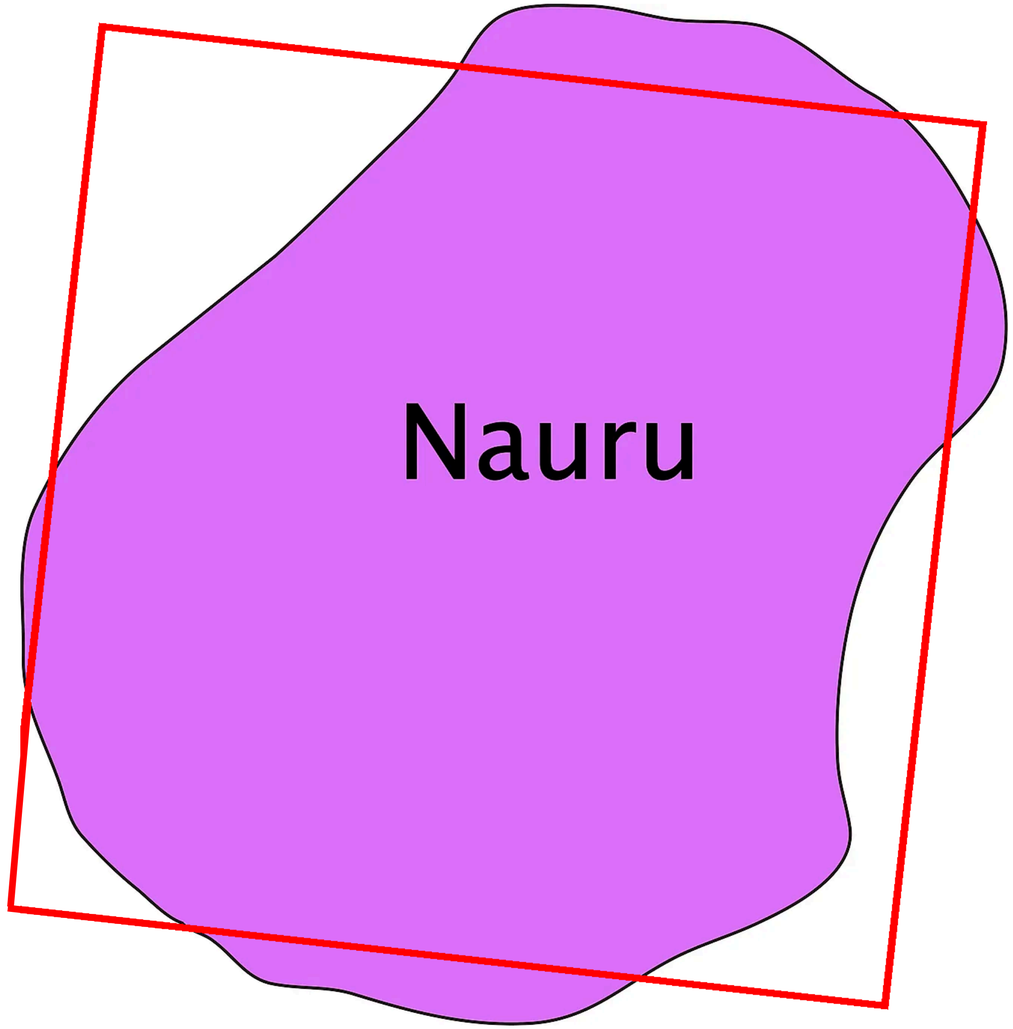
Countries that are quite square

Measuring Squareness

- Choose a square
- Measure the total error
- Divide by the area of the square
- Pick the best of all possible squares
- Take less time than the age of the universe!

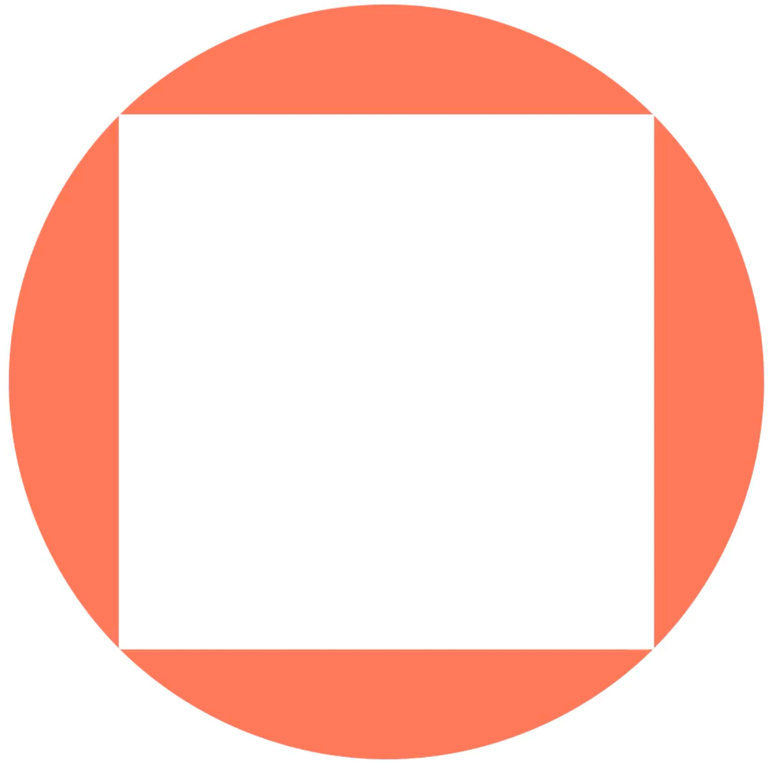
Too round?

- Nauru is “nearly a circle”
- But is being a circle actually bad?



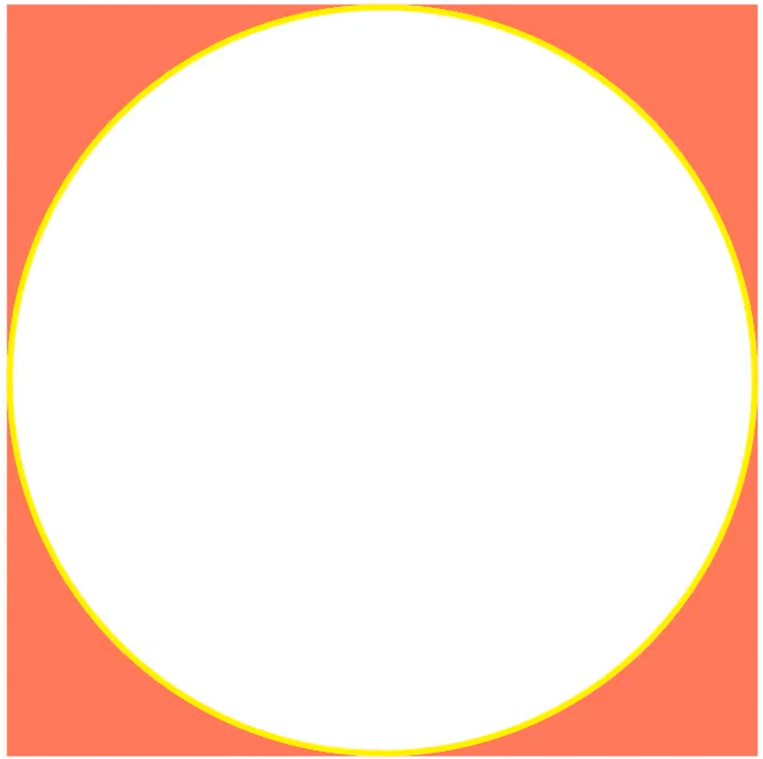
How square is a circle?

- Pick side length of $2x$
- Total error is $2\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \left| 1 - \frac{x}{\cos^2\theta} \right| d\theta$
- Then divide by $4x^2$
- Then take the derivative with respect to x
- Good luck!



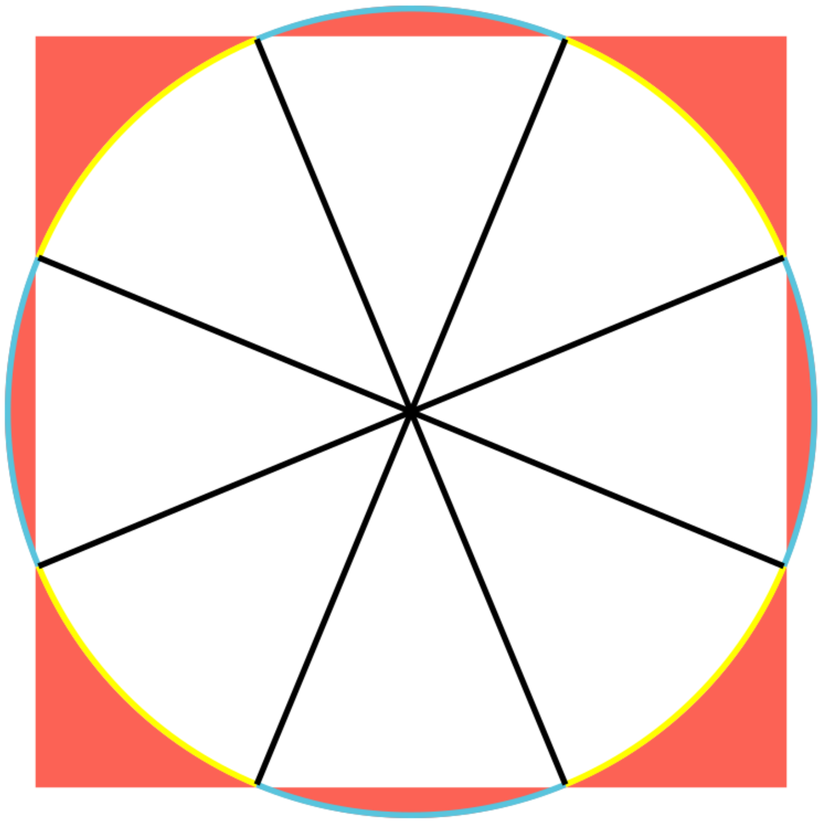
A new plan

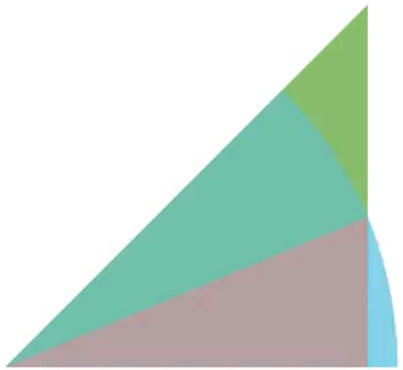
- Rescale every frame so the square is constant size
- Try to minimise the red
- Yellow arcs are where we want a bigger circle
- Blue arcs want a smaller circle

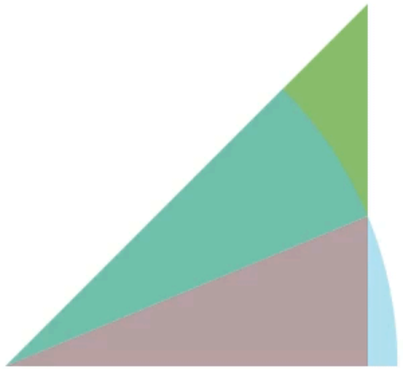


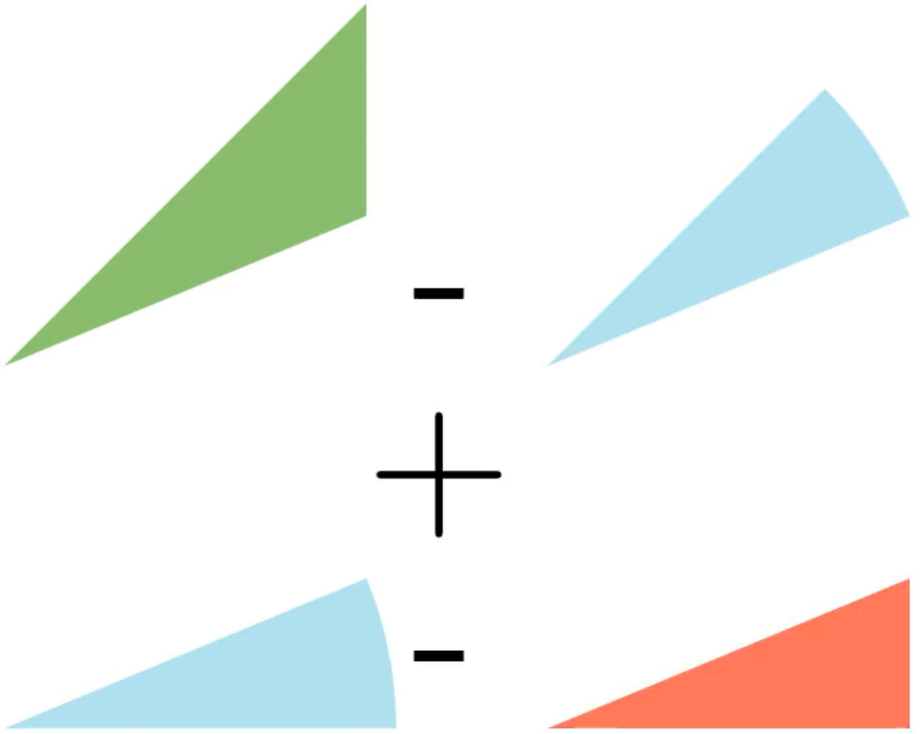
A new plan

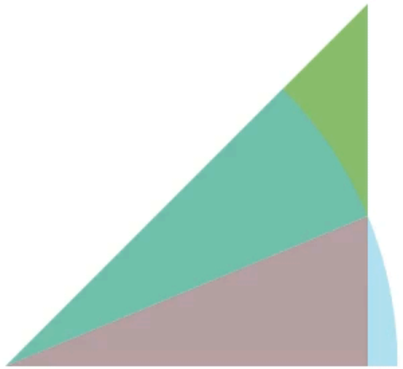
- Error is minimised when yellow length equals blue length
- ie when the angles are all 45°

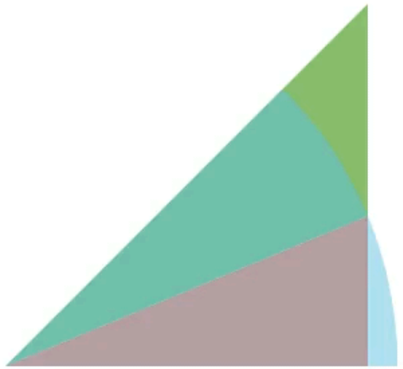


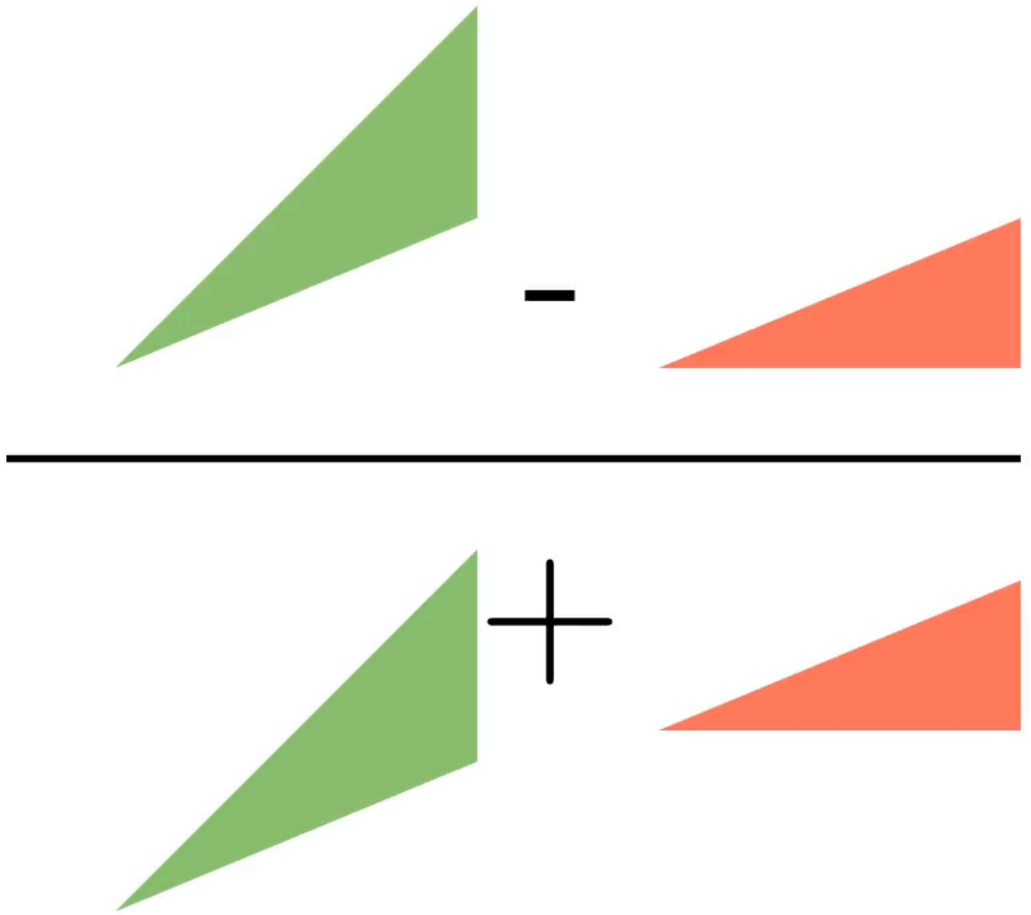


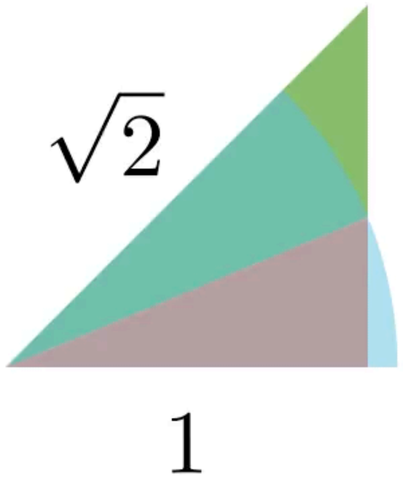


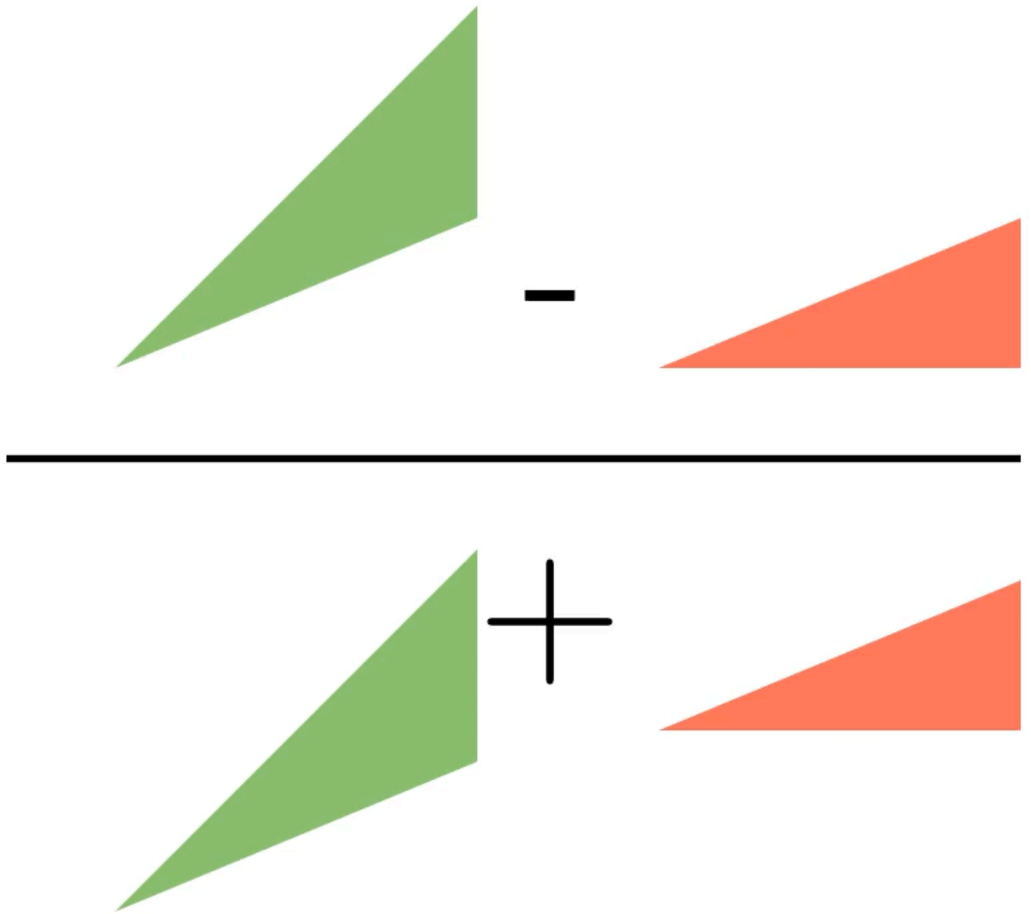









$\sqrt{2}$
1



$= \sqrt{2}$

$= 1$

$$\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$= \sqrt{2}$

$= 1$

$$3 - 2\sqrt{2}$$

$$\text{Squareness} = 2\sqrt{2} - 2$$

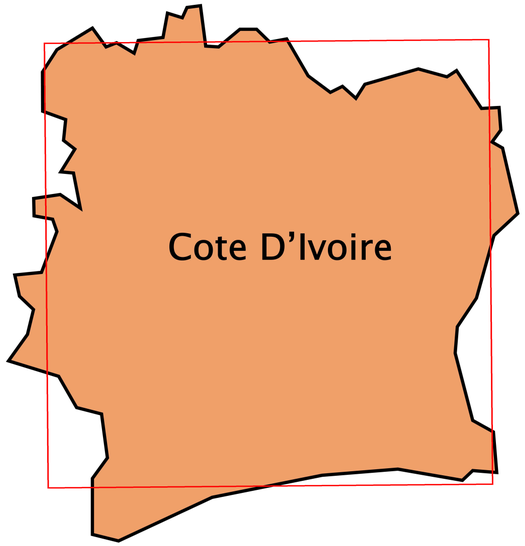
$$\approx 0.828$$

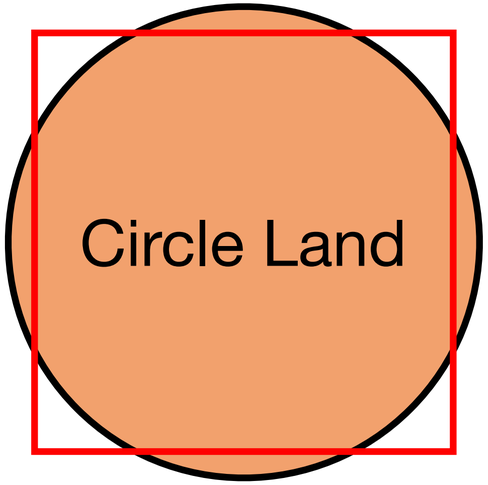
Which countries are squarer than a circle?

89.5%

83.4%

83.3%





82.8%